South County Secondary School
AP Calculus BC
Summer Assignment

<u>For students entering Calculus BC in the Fall of 2022</u>
This packet will be gone over during the first week.
The review material covered in this assignment will be quizzed with the new material sometime during the first month of school.
Please do not wait until the last minute to begin the assignment or to receive clarification of the assignment. You are now an AP student. Lesson #1 - Do not procrastinate!

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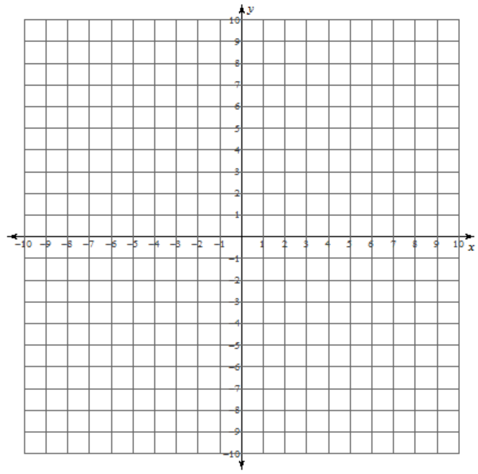
FUNCTIONS

1. Given **f(x) = -| x + 3| - 2**



Sketch

a. Domain: _____

b. Range: ______

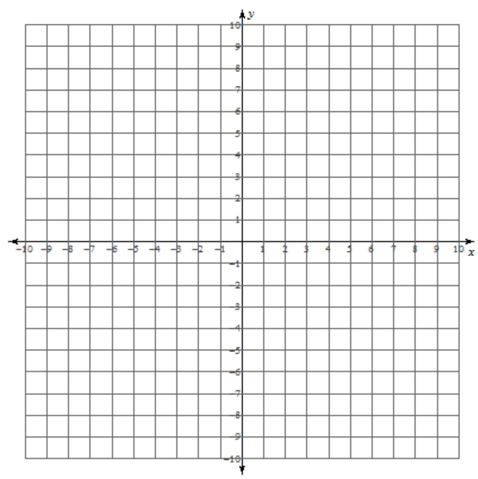
c. f(3) = ____

d. f(x+5) = _____

e. f(x) = -3 then x = ____

2. a. Graph the piece-wise function (scale the ***x-axis*** by ½-- each tick mark is worth ½)

$$g(x)=\begin{cases} x^2, & x<0 \\ \sqrt{x}, & 0<x<4 \\ \dfrac{x}{2}, & x\geq 4 \end{cases}$$



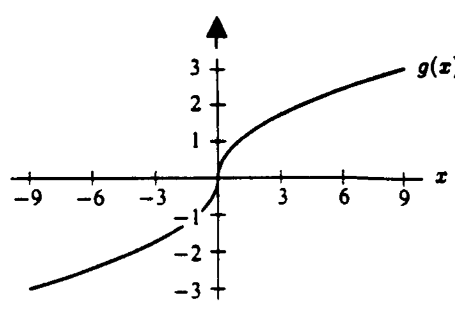
b. g(-3) = ____

c. g(1) = ____

d. g(0) = ____

e. Is g(x) a continuous function? ____

3. Given the graph of g(x) on the right



a. Estimate $\dfrac{g(6) - g(0)}{6 - 0}$ =

b. The ratio in part (a) is the slope of a line segment joining two points on the graph. Sketch this line segment.

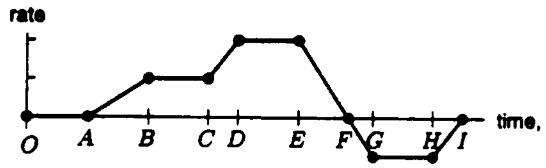
4. The rate at which water is entering a tank ($t > 0$) is represented by the given graph. A negative rate means that water is leaving the tank. State the interval(s) on which each of the following holds true:

a. The volume of water is constant.

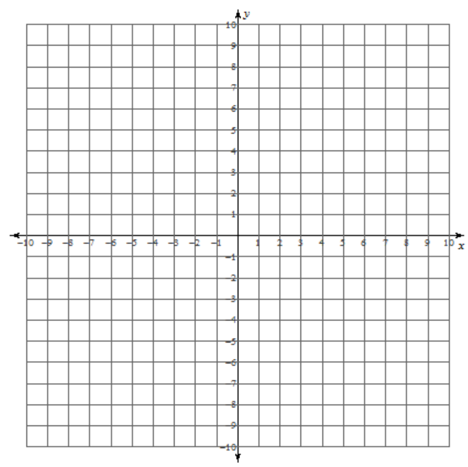
b. The volume of water is decreasing.

c. The volume of water is increasing.

d. The volume of water is increasing fastest.



5. $\mathbf{Q(x)} = \dfrac{3\mathbf{x}}{\mathbf{x}+1}$



a. Where is this function discontinuous? ___

b. State the equation of the vertical asymptote **x** = ___

c. State the equation of the horizontal asymptote **y** =___

d. Sketch the graph to the right.

e. Write the equation of the inverse of Q(x) (Switch the x & y and then rewrite as y =)

6. Use these functions: $f(x) = x+1$ $g(x) = x^2 + 2x - 3$ $h(x) = 2x - 5$

a) $h\left(\dfrac{7}{2}\right) =$ b) $g(5) =$ c) $f(x) - 4h(x) =$

d) $g(-2) =$ e) $g(h(1)) =$ f) $g(f(x)) =$

7. Simplify (no negative exponents, a single fraction):

a) $\dfrac{x^3 - 9x}{x^2 - 7x + 12}$ b) $\dfrac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$ c) $\dfrac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$ d) $\dfrac{9 - x^{-2}}{3 + x^{-1}}$

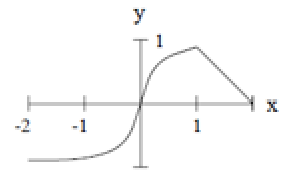
8. Rationalize the denominator:

a) $\dfrac{2}{\sqrt{3} + \sqrt{2}}$ b) $\dfrac{4}{1 - \sqrt{5}}$

9. Simplify $\dfrac{f(x+h)-f(x)}{h}$, where

a) $f(x)=2x+3$ b) $f(x)=\dfrac{1}{x+1}$ c) $f(x)=x^2-3x+1$

10. The graph of the function y = f(x) is given as follows:
Determine the graphs of the functions:
a) $f(x+1)$ b) $f(-x)$ c) $|f(x)|$ d) $f(|x|)$



TRIGONOMETRY

What you need to know:

- Trig functions and inverse trig functions for all special angles (unit circle)
- Fundamental trig identities (reciprocal, quotient, Pythagorean)
- Graphs of sin x, cos x, tan x
- Domain and range of sin x, cos x, tan x
- How to solve trig equations

1. Evaluate without use of a calculator.

(a) $\tan(\frac{\pi}{6})$
(b) $\cos(\frac{-\pi}{3})$
(c) $\sin(\pi)$
(d) $\csc(\frac{\pi}{2})$
(e) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$
(f) $\cos^{-1}\left(\frac{1}{2}\right)$
(g) $\arctan(-\sqrt{3})$
(h) $\sec^{-1}(-2)$
(i) $\arcsin(-\frac{\sqrt{2}}{2})$
(j) $\sin(\cos^{-1}0.6)$

2. Sketch the following graphs without the use of a calculator. Show at least two periods and mark your axes.

a) $\sin\left(x-\frac{\pi}{4}\right)$ b) $\sin\left(\frac{x}{2}\right)$ c) $2\sin(x)$ d) $\dfrac{1}{\sin x}$ e) $y=2\tan\left(x-\frac{\pi}{4}\right)$

EXPONENTS

SIMPLIFY COMPLETELY:

1. $2\left(x^4y^3\right)^0$

2. $\dfrac{8^{-2} * 4^{-2}}{16^{-2}}$

3. $\dfrac{3c^2d^3}{\left(3cd^{-2}\right)^2}$

4. $\dfrac{3^7 * 9^5}{\sqrt{27^{12}}}$

5. $(32)^{-2/5}$

6. $\sqrt{x} * \sqrt[3]{x} * \sqrt[4]{x}$

7. $\dfrac{x^2 - x + 7}{x}$

8. $\dfrac{x^3 - x + 1}{\sqrt{x}}$

9. $x^{3/2}\left(x^2 - 2x\right)$

LOGARITHMS & EXPONENTIALS

Solve for x without a calculator. (#8 & 9 are calc).

1. $\dfrac{1}{3} = 3^{2x+2}$

2. $\log_{\frac{1}{2}} x = 4$

3. $\log_3 81 = x$

4. $\log_3(-9) = x$

5. $\log_x 16 = -4$

6. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

7. $2^x = 3$

8. $2.43 \bullet 10^x = 1.84$

9. $\ln(x+5) = -\ln(x-1) - \ln(x+1)$

SOLVING EQUATIONS

1. Solve the following equations for the indicated variable.

a) $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$, solve for *a*

b) $A = 2\pi r^2 + 2\pi rh$, solve for positive *r*

c) $V = 2\left(ab + bc + ca\right)$, solve for a

SOLVE FOR X.

1. $15 + x - 2x^2 = 0$

2. $(x+2)^{3/4} = 27$

3. $\sqrt{x-2} - 8 = 0$

4. $\dfrac{1}{x-2} = 3$

5. $4t^3 - 12t^2 + 8t - 24 = 0$

6. $\dfrac{4}{x-3} - \dfrac{4}{x} = 1$

7. $\left|x - \dfrac{3}{2}\right| \le 1$

8. $e^{2x} - 7e^x + 10 = 0$

9. $\left|x^2 - 6\right| = x$

10. $2\cos x + 1 = 0$

11. $3e^{-5x} = 132$

12. $\ln x - \ln 3 = 2$

13. $3\sec^2 x - 4 = 0$

14. $\sin^2 x = 3\cos^2 x$

15. $x^2 + 4x + 3 = 0$ solve by completing the square

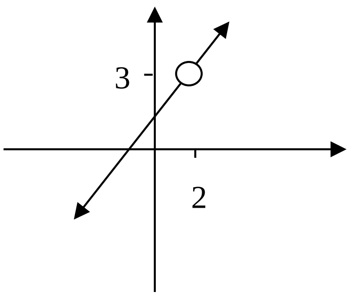
Determine the equation of the following lines (leave in point-slope form).
16. line through (-1, 3) and (2, -4)

17. the line through (-1, 2) and perpendicular to the line $2x - 3y + 5 = 0$

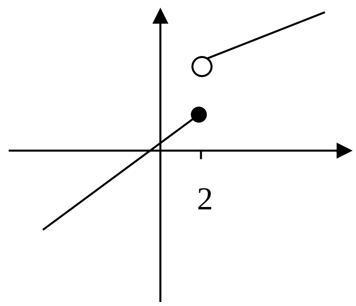
18. The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution x = 2. Find all other solutions.

LIMITS

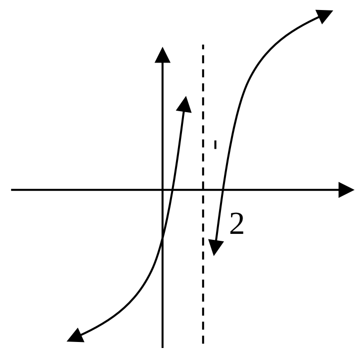
The limit of a function is the y-value that you are getting close to as x gets close to some number in the domain. In the "limit process" you never get to the limit, except for the limit of a constant function. We write $\lim_{x \to a} f(x)$, which is read "the limit of f(x) as x approaches a domain value of a." The limit must be the same as x approaches "a" from both the left and the right. To find the limit, substitute in values very close to "a" on both left and right and see if the y-value is approaching a single value. The limit does exist at a hole in a graph, but does not exist at a vertical asymptote or a jump in the graph.



Limit exist at 2



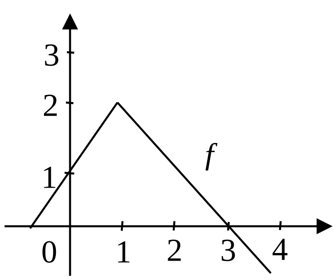
No limit at 2



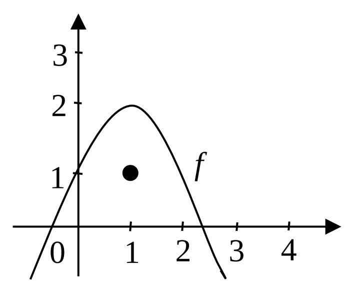
No limit at 2

The graphs of some functions are pictured below. Do you think that $\lim_{x \to 1} f(x)$ exists? If you think the limit does exits, state its value.

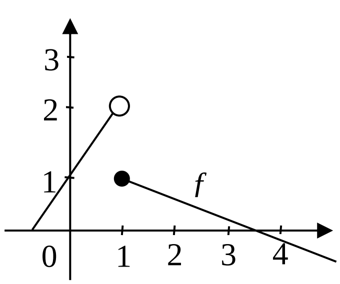
1.



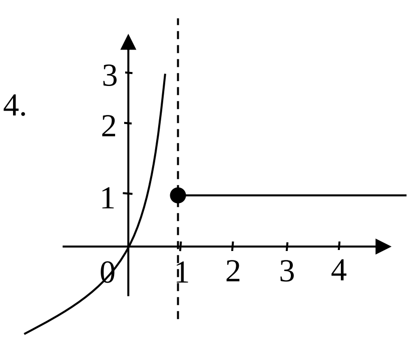
2.



3.



4.



State the value of each of the following:

5. $\lim_{x \to 3} 5$

6. $\lim_{x \to -2} \frac{x^2 - 4}{x^3 + 8}$

7. $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

8. $\lim_{x \to 0} \frac{\sqrt{9 + x} - 3}{x}$

9. $\lim_{x \to 0} \frac{\sin x}{3x}$

10. $\lim_{x \to -2} |x|$

DERIVATIVES

Find the derivative using the definition of derivative.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

1. $f(x) = 2x$

2. $f(x) = x^2$

3. $f(x) = 1/x$

Find the derivatives using the power rule.

4. $f(x) = x^3$

5. $f(x) = \sqrt{x}$

6. $f(x) = 6x^3 - 2x$

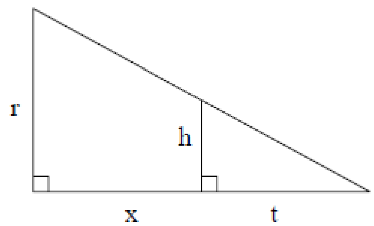
7. $f(x) = x^{4/3}$

8. $f(x) = x^{-4}$

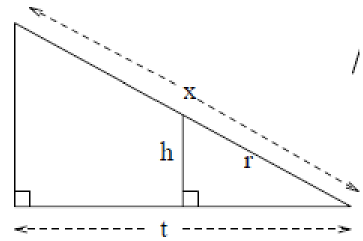
9. $f(x) = \frac{1}{x^5}$

Word Problems

1) Express x in terms of the other variable in the picture.

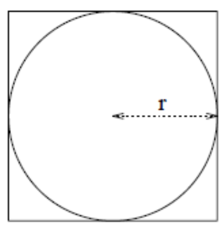


(a)

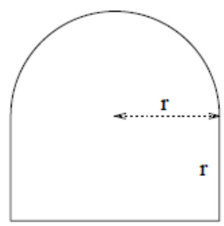


(b)

2) a) Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below.



(a)



(b)

c) Find a formula for the perimeter of a window of the shape in the picture (b) above.

3) A water tank has the shape of a cone (like an ice cream cone without ice cream). The tank is 10m high and has a radius of 3m at the top. If the water is 5m deep (in the middle), what is the surface area of the top of the water?

4) Two cars start moving from the same point. One travels south at 100 km/hour, the other west at 50 km/hour. How far apart are they two hours later?

5) A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal? (Assume that the string is perfectly straight).